Friedmann equations from emergence of cosmic space

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Padmanabhan [arXiv:1206.4916] argues that the cosmic acceleration can be understood from the perspective that spacetime dynamics is an emergence phenomena. By calculating the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space, he also arrived at Friedmann equation in flat universe. In this paper, by modification his proposal, we are able to derive the Friedmann equation of the Friedmann-Robertson-Walker (FRW) Universe with any spatial curvature. We also extend the study to higher dimensional spacetime and derive successfully the Friedmann equations not only in Einstein gravity, but also in Gauss-Bonnet and more general Lovelock gravity with any spacial curvature. This is the first derivation of Friedmann equations in these gravity theories in a nonflat FRW Universe by using the novel idea proposed by Padmanabhan. Our study indicates that the approach presented here is enough powerful and further supports the viability of the Padmanabhan's perspective of emergence gravity.

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I. INTRODUCTION

Physicists have been speculating on the nature and origin of gravity for a long time. Newton believed that gravity is just a force like other forces of the nature and does not affect on the space. This was a general belief until Einstein presented his theory of general relativity in 1915. According to Einstein's theory, gravity is just the spacetime curvature. In this new picture, the matter field tells space (geometry) how to curve, and the geometry tells matter how to move. Also, according to the equivalence principle of general relativity, gravity is just the dynamics of spacetime. This implies that gravity is an emergent phenomenon.

In $1970's$ thermodynamics of black holes were studied. According to laws of black holes mechanics, a black hole can be regarded as a thermodynamical system which has temperature proportional to its surface gravity and an entropy proportional to its horizon area. This indicates that geometrical quantities such as horizon area and surface gravity are closely related to the thermodynamic quantities like temperature and entropy. Are there a direct connection between gravitational field equations describing the geometry of spacetime and the first law of thermodynamics? Jacobson [1] was indeed the first who answered this question by disclosing that the Einstein field equations can be derived by applying the Clausius relation $\delta Q = T\delta S$ on the horizon of spacetime, here δS is the change in the entropy and δQ and T are, respectively, the energy flux across the horizon and the Unruh temperature seen by an accelerating observer just inside the horizon.

The next great step toward understanding the nature of gravity put forwarded by Verlinde [2] in 2010 who claimed that gravity is not a fundamental interaction but should be interpreted as an entropic force caused by changes of entropy associated with the information on the holographic screen. Applying the first principles, namely the holographic principle and the equipartition law of energy, Verlinde derived the Newton's law of gravitation, the Poisson equation and in the relativistic regime the Einstein field equations. Although in [3] Padmanabhan observed that the equipartition law of energy for the horizon degrees of freedom combining with the thermodynamic relation $S = E/2T$, leads to the Newton's law of gravity, however, the idea that gravity is not a fundamental force and can be interpreted as the entopic force was first pointed by Verlinde [2]. Following [2], some attempts have been done to investigate the entropic origin of gravity in different setups (see [6–13] and references therein). Nevertheless, there are some critical comments on Verlinde's proposal [4]. Strong criticism against the entropic origin of gravity was presented by Visser [5] who claimed that the interpretation of gravity as an entropic force is untenable. According to Visser arguments [5], if one would like to reformulate classical Newtonian gravity in terms of an entropic force, then the fact that Newtonian gravity is described by a conservative force places significant constraints on the form of the entropy and temperature functions.

Although Verlinde's proposal has changed our understanding on the origin and nature of gravity, but it considers the gravitational field equations as the equations of emergent phenomenon and leave the spacetime as a background geometric which has already exist. Is it possible to regard the spacetime itself as an emergent structure? Recently, by calculating the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space, Padmanabhan [14] argued that spacetime dynamics can be emerged. As a result, he is able to explain the origin of the acceleration of the universe expansion from his new perspective [14]. According to Padmanabhan, the spatial expansion of our universe can be regarded as the consequence of emergence of space

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and *the cosmic space is emergent as the cosmic time progresses*. Using this new idea, Padmanabhan [14] derived the Friedmann equation of a flat FRW Universe. Following [14], Cai obtained the Friedmann equation of a higher dimensional FRW Universe in Einstein, Gauss- Bonnet and Lovelock theory [15]. Similar derivation were also made by the authors of [16]. Instead of modifying the number of degrees of freedom on the holographic surface of the Hubble sphere, and the volume increase, the authors of [16], assumed that (dV/dt) is proportional to a function $f(\triangle N)$. Here $\triangle N = N_{\rm sur} - N_{\rm bulk}$, where $N_{\rm sur}$ is the number of degrees of freedom on the boundary and $N_{\rm bulk}$ is the number of degrees of freedom in the bulk. When the volume of the spacetime is constant, the function $f(\triangle N)$ is equal to zero. It is worth mentioning that the authors of [15, 16] only derived the Friedmann equations of the spatially flat FRW Universe in Gauss-Bonnet and Lovelock gravities, and failed to arrive at Friedmann equations with any spacial curvature in these gravity theories. For this purpose, they proposed the Hawking temperature associated with the Hubble horizon to be $T = H/2\pi$ and the volume of the universe is $V = 4\pi H^{-3}/3$.

In this paper, by modifying the original proposal of Padmanabhan [14], we are able to derive the Friedmann equation of the FRW Universe with any spacial curvature. Note that in a nonflat universe, the Hawking temperature and the volume are usually taken as $T = 1/2\pi\tilde{r}_A$ and $V = 4\pi\tilde{r}_A^3/3$, respectively, where $\tilde{r}_A$ is the apparent horizon radius [17]. We also generalize the study to the higher dimensional spacetime and higher order gravities, and derive the corresponding dynamical equations governing the evolution of the universe with any spacial curvature not only in Einstein gravity, but also in Gauss-Bonnet and more general lovelock gravity. For consistency, in all cases we set the integration constant equal to zero. In the next section we extract the Friedmann equation by properly modifying the proposal of [14]. In section III, we extend our study to higher order gravity theory in arbitrary dimension. We summarize our results in section IV.

II. FRIEDMANN EQUATION IN 4D EINSTEIN GRAVITY

We assume the background spacetime is spatially homogeneous and isotropic which is described by the line element

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $\tilde{r} = a(t)r$, $x^0 = t, x^1 = r$, the two dimensional metric is h_{ab}=diag $(-1, a^2/(1-kr^2))$. Here k denotes the curvature of space with $k = 0, 1, -1$ corresponding to flat, closed, and open universes, respectively. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$. For a dynamical spacetime, the apparent horizon has been argued to be a causal horizon and is associated with the gravitational entropy and surface gravity [18]. A simple calculation gives the apparent horizon radius for the FRW Universe as [19]

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter. It is widely accepted that the apparent horizon is a suitable boundary of our universe, from thermodynamic viewpoint, for which all laws of thermodynamics are hold on it. Thermodynamical properties of the apparent horizon has been studied in different setups [20–22]. Following [14], we assume the number of degrees of freedom on the spherical surface of apparent horizon with radius $\tilde{r}_A$ is proportional to its area and is given by

$$N_{\rm sur} = 4S = \frac{4\pi\tilde{r}_A^2}{L_p^2}, \tag{3}$$

where L_p is the Planck length, $A = 4\pi\tilde{r}_A^2$ represents the area of the apparent horizon and S is the entropy which obeys the area law. Assume the temperature associated with the apparent horizon is the Hawking temperature [17]

$$T = \frac{1}{2\pi\tilde{r}_A}, \tag{4}$$

and the energy contained inside the sphere with volume $V = 4\pi\tilde{r}_A^3/3$ is the Komar energy

$$E_{\rm Komar} = |(\rho + 3p)|V. \tag{5}$$

According to the equipartition law of energy, the bulk degrees of freedom obey

$$N_{\rm bulk} = \frac{2|E_{\rm Komar}|}{T}. \tag{6}$$

Through this paper we set $k_B = 1 = c = \hbar$ for simplicity. The novel idea of Padmanabhan is that the cosmic expansion, conceptually equivalent to the emergence of space, is being driven towards holographic equipartition, and the basic law governing the emergence of space must relate the emergence of space to the difference between the number of degrees of freedom in the holographic surface and the one in the emerged bulk [14]. He proposed that in an infinitesimal interval dt of cosmic time, the increase dV of the cosmic volume, in flat universe, is given by

$$\frac{dV}{dt} = L_p^2(N_{\rm sur} - N_{\rm bulk}). \tag{7}$$

In general, one may expect dV/dt to be some function of $(N_{\rm sur} - N_{\rm bulk})$ which vanishes when the latter does. In this case one may regard Eq. (7) as a Taylor series expansion of this function truncated at the first order [14]. This approach was studied recently [16].

Motivated by (7), we propose the volume increase, in a nonflat FRW Universe, is still proportional to the difference between the number of degrees of freedom on the apparent horizon and in the bulk, but the function of proportionality is not just a constant, and it equals to the ratio of the apparent horizon and Hubble radius. Therefore we write down

$$\frac{dV}{dt} = L_p^2 \frac{\tilde{r}_A}{H^{-1}} (N_{\rm sur} - N_{\rm bulk}). \tag{8}$$

It is well known that for pure de Sitter spacetime the number of degrees of freedom in a bulk and the number of degrees of freedom on the boundary surface are equal, namely $N_{\rm sur} = N_{\rm bulk}$ [14]. Since our universe, is not exactly de Sitter but it is asymptotically de Sitter, thus for our universe, Padmanabhan proposed [14]

$$\frac{dV}{dt} \propto (N_{\rm sur} - N_{\rm bulk}). \tag{9}$$

In order to arrive at the desired dynamical equations for the FRW Universe, in Einstein gravity, he assumed the constant of proportionality to be L_p^2. For a nonflat Universe and other gravity theories, the assumption (7) does not work and we found out that it should be modified as in Eq. (8). One may regard the assumption (8) to the status of a postulate and verify whether it can lead to the correct Friedmann equations describing the evolution of the Universe. In this paper, we will show that with this modification, we are able to extract the Friedmann equations with any spacial curvature in Einstein, Gauss-Bonnet and more general Lovelock gravity. This may justify the correctness of our assumption in (8). For spatially flat universe, $\tilde{r}_A = H^{-1}$, and one recovers the proposal (7).

Taking the time derivative of the cosmic volume $V = 4\pi\tilde{r}_A^3/3$, we have

$$\frac{dV}{dt} = 4\pi\tilde{r}_A^2 \dot{\tilde{r}}_A. \tag{10}$$

Substituting the cosmic volume V and the temperature (4) in Eq. (6), we find the numbers of degrees of freedom in the bulk as

$$N_{\rm bulk} = -\frac{16\pi^2}{3}(\rho + 3p)\tilde{r}_A^4. \tag{11}$$

In order to have $N_{\rm bulk} > 0$, we take $\rho + 3p < 0$ [14]. Substituting Eqs. (3), (10), (11) into (8), we arrive at

$$4\pi\tilde{r}_A^2 \dot{\tilde{r}}_A = L_p^2 \frac{\tilde{r}_A}{H^{-1}} \left[\frac{4\pi\tilde{r}_A^2}{L_p^2} + \frac{16\pi^2}{3}(\rho + 3p)\tilde{r}_A^4\right]. \tag{12}$$

Rearranging the terms we obtain

$$4\pi\tilde{r}_A^2 \left(\dot{\tilde{r}}_A H^{-1} - \tilde{r}_A\right) = \frac{16\pi^2 L_p^2}{3}(\rho + 3p)\tilde{r}_A^5, \tag{13}$$

which can be simplified as

$$\tilde{r}_A^{-3}(\dot{\tilde{r}}_A H^{-1} - \tilde{r}_A) = \frac{4\pi L_p^2}{3}\left[3(\rho + p) - 2\rho\right]. \tag{14}$$

Using the continuity equation, $\dot{\rho} + 3H(\rho + p) = 0$, we reach

$$\tilde{r}_A^{-3}(\dot{\tilde{r}}_A H^{-1} - \tilde{r}_A) = -\frac{4\pi L_p^2}{3}\left[\dot{\rho}H^{-1} + 2\rho\right]. \tag{15}$$

Multiplying the both hand side of (15) by factor $2\dot{a}a$, and using the fact that $H^{-1} = a/\dot{a}$, we get

$$2\dot{a}a\tilde{r}_A^{-2} - 2a^2\dot{\tilde{r}}_A\tilde{r}_A^{-3} = \frac{8\pi L_p^2}{3}\left[\dot{\rho}a^2 + 2\rho\dot{a}a\right]. \tag{16}$$

The above equation can be further rewritten as

$$\frac{d}{dt}\left(a^2\tilde{r}_A^{-2}\right) = \frac{d}{dt}\left[a^2\left(H^2 + \frac{k}{a^2}\right)\right] = \frac{8\pi L_p^2}{3}\frac{d}{dt}(\rho a^2), \tag{17}$$

where we have also used relation (2). Integrating, we obtain

$$H^2 + \frac{k}{a^2} = \frac{8\pi L_p^2}{3}\rho, \tag{18}$$

where we have set the integration constant equal to zero. In this way we derive the Friedmann equation of the FRW Universe with any spacial curvature, by calculating the difference between the number of degrees of freedom in the bulk and on the apparent horizon. Let us stress here the difference between our derivation and ones presented in [15, 16]. The authors of [15, 16] arrived at (18), by using proposal [14] given by Eq. (7), and interpreting the integration constant as the special curvature, while we arrive at the same result by modifying the proposal of [14] in the form of (8), and setting the integration constant equal to zero.

III. FRIEDMANN EQUATION IN GAUSS-BONNET AND LOVELOCK GRAVITY

In this section, we apply the approach developed in the previous section to derive the Friedmann equations in Gauss-Bonnet and more general Lovelock gravity with any spacial curvature. This is the first derivation of Friedmann equations in these gravity theories in a nonflat FRW Universe by using the novel idea presented in [14]. We first extend the approach of the previous section to the $(n+1)$-dimensional spacetime. In this case the number of degrees of freedom on the apparent horizon turn out to be [15]

$$N_{\rm sur} = \alpha\frac{A}{L_p^2}, \tag{19}$$

where $A = n\Omega_n\tilde{r}_A^{n-1}$ and $\alpha = (n-1)/2(n-2)$, with Ω_n is the volume of an unit n-sphere. We also modify our proposal in (8) a little as

$$\alpha\frac{dV}{dt} = L_p^{n-1}\frac{\tilde{r}_A}{H^{-1}}(N_{\rm sur} - N_{\rm bulk}), \tag{20}$$

where the volume of the n-sphere is $V = \Omega_n \tilde{r}_A^n$. The bulk Komar energy in $(n+1)$-dimensions is given by [23]

$$E_{\rm Komar} = \frac{(n-2)\rho + np}{n-2} V, \tag{21}$$

and hence the bulk degrees of freedom is obtained as

$$N_{\rm bulk} = -4\pi\Omega_n \tilde{r}_A^{n+1} \frac{(n-2)\rho + np}{n-2}, \tag{22}$$

where we take $(n-2)\rho + np < 0$ in order to have $N_{\rm bulk} > 0$ [14]. Substituting Eqs. (19) and (22) in relation (20), one gets

$$\tilde{r}_A^{-2} - \dot{\tilde{r}}_A H^{-1} \tilde{r}_A^{-3} = -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho + np]. \tag{23}$$

Multiplying the both hand side by factor $2\dot{a}a$, after using the continuity equation in $(n+1)$-dimensions as

$$\dot{\rho} + nH(\rho + p) = 0, \tag{24}$$

we arrive at

$$\frac{d}{dt}\left[a^2\left(H^2 + \frac{k}{a^2}\right)\right] = \frac{16\pi L_p^{n-1}}{n(n-1)} \frac{d}{dt}(\rho a^2). \tag{25}$$

Integrating, we find

$$H^2 + \frac{k}{a^2} = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho, \tag{26}$$

where we have set the integration constant equal to zero. This is the Friedmann equation of $(n+1)$-dimensional FRW Universe with any spacial curvature [17].

Up to now we only considered Einstein gravity, and derive the corresponding Friedmann equations in a Universe with spacial curvature. Now we want to see whether the above procedure works or not in other gravity theories such as the Gauss-Bonnet and more general Lovelock gravity. Lovelock gravity is the most general lagrangian which keeps the field equations of motion for the metric of second order, as the pure Einstein-Hilbert action [24]. Let us first consider the Gauss-Bonnet theory. The key point which should be noticed here is that in Gauss-Bonnet gravity the entropy of the holographic screen does not obey the area law. Static black hole solutions of Gauss-Bonnet gravity have been found and their thermodynamics have been investigated in ample details [25, 26]. The entropy of the static spherically symmetric black hole in Gauss-Bonnet theory has the following expression [26]

$$S = \frac{A_+}{4L_p^{n-1}}\left[1 + \frac{n-1}{n-3}\frac{2\tilde{\alpha}}{r_+^2}\right], \tag{27}$$

where $A_+ = n\Omega_n r_+^{n-1}$ is the horizon area and r_+ is the horizon radius. In the above expression $\tilde{\alpha} = (n-2)(n-3)\alpha$, where α is the Gauss-Bonnet coefficient which is positive [25]. For $n = 3$ we have $\tilde{\alpha} = 0$, thus the Gauss-Bonnet correction term contributes only for $n \geq 4$. We assume the entropy expression (27) also holds for the apparent horizon of the FRW Universe in Gauss-Bonnet gravity. The only change we need to apply is the replacement of the horizon radius r_+ with the apparent horizon radius $\tilde{r}_A$, namely

$$S = \frac{A}{4L_p^{n-1}}\left[1 + \frac{n-1}{n-3}\frac{2\tilde{\alpha}}{\tilde{r}_A^2}\right], \tag{28}$$

where $A = n\Omega_n \tilde{r}_A^{n-1}$ is the apparent horizon area. We define the effective area of the holographic surface corresponding to the entropy (28) as

$$\widetilde{A} = n\Omega_n \tilde{r}_A^{n-1}\left[1 + \frac{n-1}{n-3}\frac{2\tilde{\alpha}}{\tilde{r}_A^2}\right]. \tag{29}$$

Now we calculate the increasing in the effective volume as

$$\frac{d\widetilde{V}}{dt} = \frac{\tilde{r}_A}{(n-1)}\frac{d\widetilde{A}}{dt} = n\Omega_n \dot{\tilde{r}}_A \tilde{r}_A^{n-1}(1 + 2\tilde{\alpha}\tilde{r}_A^{-2}) \tag{30}$$

$$= -\frac{n\Omega_n \tilde{r}_A^{n+2}}{2}\frac{d}{dt}\left(\tilde{r}_A^{-2} + \tilde{\alpha}\tilde{r}_A^{-4}\right). \tag{31}$$

Inspired by (31), we propose that the number of degrees of freedom on the apparent horizon, in Gauss-Bonnet gravity, is given by

$$N_{\rm sur} = \frac{\alpha n\Omega_n \tilde{r}_A^{n+1}}{L_p^{n-1}}\left(\tilde{r}_A^{-2} + \tilde{\alpha}\tilde{r}_A^{-4}\right). \tag{32}$$

The bulk degrees of freedom is still given by (22). Inserting Eqs. (22), (30) and (32) in relation (20), with replacing $V \to \tilde{V}$, we obtain

$$(\tilde{r}_A^{-2} + \tilde{\alpha}\tilde{r}_A^{-4}) - \dot{\tilde{r}}_A H^{-1}\tilde{r}_A^{-3}(1 + 2\tilde{\alpha}\tilde{r}_A^{-2}) \tag{33}$$

$$= -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho + np]. \tag{34}$$

Multiplying the both hand side of (34) by factor $2\dot{a}a$, with help of continuity equation (24) and relation (2), we get

$$\frac{d}{dt}\left\{a^2\left[H^2 + \frac{k}{a^2} + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)^2\right]\right\} = \frac{16\pi L_p^{n-1}}{n(n-1)}\frac{d}{dt}(\rho a^2). \tag{35}$$

Integrating, we find

$$H^2 + \frac{k}{a^2} + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)^2 = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho, \tag{36}$$

where again we have set the integration constant equal to zero. This is indeed, the corresponding Friedmann equation of the FRW Universe with any spacial curvature in Gauss-Bonnet gravity [17]. Note that the authors of Refs. [15, 16] could derive the above equation only in

a flat FRW Universe, while we derive it with arbitrary spacial curvature. This may show the viability of our proposal (20).

Finally, we consider the more general Lovelock gravity. The entropy of the spherically symmetric black hole solutions in Lovelock theory can be expressed as [27]

$$S = \frac{A_+}{4L_p^{n-1}} \sum_{i=1}^{m} \frac{i(n-1)}{(n-2i+1)} \hat{c}_i {r_+}^{2-2i}, \qquad (37)$$

where $m = [n/2]$ and the coefficients $\hat{c}_i$ are given by

$$\hat{c}_0 = \frac{c_0}{n(n-1)}, \quad \hat{c}_1 = 1, \quad \hat{c}_i = c_i \prod_{j=3}^{2m} (n+1-j) \quad i > 1. \qquad (38)$$

We further assume the entropy expression (37) are valid for a FRW Universe bounded by the apparent horizon in the Lovelock gravity provided we replace the horizon radius r_+ with the apparent horizon radius $\tilde{r}_A$, namely

$$S = \frac{A}{4L_p^{n-1}} \sum_{i=1}^{m} \frac{i(n-1)}{(n-2i+1)} \hat{c}_i \tilde{r}_A^{2-2i}. \qquad (39)$$

It is easy to show that, the first term in the above expression leads to the well known area law. The second term yields the apparent horizon entropy in Gauss-Bonnet gravity. We suppose from the entropy expression that the effective area of the apparent horizon in Lovelock gravity is given by

$$\widetilde{A} = n\Omega_n \tilde{r}_A^{n-1} \sum_{i=1}^{m} \frac{i(n-1)}{(n-2i+1)} \hat{c}_i \tilde{r}_A^{2-2i}, \qquad (40)$$

and the increase of the effective volume is then given by

$$\frac{d\widetilde{V}}{dt} = \frac{\tilde{r}_A}{(n-1)} \frac{d\widetilde{A}}{dt} = n\Omega_n \tilde{r}_A^{n+1} \left(\sum_{i=1}^{m} i\hat{c}_i \tilde{r}_A^{-2i} \right) \dot{\tilde{r}}_A \qquad (41)$$

$$= -\frac{n\Omega_n \tilde{r}_A^{n+2}}{2} \frac{d}{dt} \left(\sum_{i=1}^{m} \hat{c}_i \tilde{r}_A^{-2i} \right). \qquad (42)$$

In this case, we assume from (42) that the number of degrees of freedom on the apparent horizon, in Lovelock gravity, is

$$N_{\rm sur} = \frac{\alpha n\Omega_n}{L_p^{n-1}} \tilde{r}_A^{n+1} \sum_{i=1}^{m} \hat{c}_i \tilde{r}_A^{-2i}. \qquad (43)$$

Substituting (22), (41) and (43) into (20), we reach

$$\sum_{i=1}^{m} \hat{c}_i \tilde{r}_A^{-2i} - \dot{\tilde{r}}_A H^{-1} \sum_{i=1}^{m} i\hat{c}_i \tilde{r}_A^{-2i-1} \qquad (44)$$

$$= -\frac{8\pi L_p^{n-1}}{n(n-1)} [(n-2)\rho + np]. \qquad (45)$$

Multiplying the both hand side by factor $2\dot{a}a$, after using the continuity equation (24) as well as definition (2), we obtain

$$\frac{d}{dt} \left[a^2 \sum_{i=1}^{m} \hat{c}_i \left(H^2 + \frac{k}{a^2} \right)^i \right] = \frac{16\pi L_p^{n-1}}{n(n-1)} \frac{d}{dt} (\rho a^2). \qquad (46)$$

After integrating and setting the constant of integration equal to zero, we find the corresponding Friedmann equation of the FRW Universe with any spacial curvature in Lovelock gravity,

$$\sum_{i=1}^{m} \hat{c}_i \left(H^2 + \frac{k}{a^2} \right)^i = \frac{16\pi L_p^{n-1}}{n(n-1)} \rho. \qquad (47)$$

This is exactly the result obtained in [17] by applying the first law of thermodynamics on the apparent horizon of the FRW Universe in Lovelock gravity. Here we arrived at the same result by using quite different approach. This indicates that, given the entropy expression at hand, one is able to reproduce the corresponding dynamical equation with any spacial curvature, by applying the proposal (20).

IV. SUMMARY AND DISCUSSION

We have investigated the novel idea recently proposed by Padmanabhan [14], which states that the emergence of space and Universe expansion can be understood by calculating the difference between the number of degrees of freedom on the Hubble horizon and the one in the emerged bulk. Applying this idea to a flat FRW Universe with Hubble horizon, he derived the dynamical equation describing the evolution of the Universe [14]. In this paper, by properly modification his idea, we derived the Friedmann equation of a FRW Universe with any spacial curvature. Our approach not only works in Einstein gravity, but also works very well in Gauss-Bonnet and more general Lovelock gravity. The key assumption here is that in a nonflat Universe, the volume increase, is still proportional to the difference between the number of degrees of freedom on the apparent horizon and in the bulk, but the function of proportionality is not just the constant L_p^2, instead it equals to the ratio of the apparent horizon radius and the Hubble radius, i.e., $L_p^2 \tilde{r}_A / H^{-1}$.

It is important to note that Padmanabhan's proposal (7) can lead to the Friedmann equation with spacial curvature only in Einstein gravity [15, 16]. The main result of the present work is that the modified proposal (8) can lead to the Friedmann equations of the FRW Universe with any spacial curvature in higher order gravity theories. Indeed, while the authors of [15, 16] interpreted the integration constant as the spatial curvature k in Einstein gravity, they failed to interpret the constant of integration as the spatial curvature in the cases of Gauss-Bonnet and Lovelock gravities. This is due to the fact that, in Einstein gravity, the de Sitter Universe can be described

either by $k = 0$ or $k = 1$. As a result, in Gauss-Bonnet and Lovelock gravity, with proposal (7), they could only derive the Friedmann equations of the flat Universe.

In summary, given the entropy expression at hand, one is able to reproduce the corresponding dynamical equation of the FRW Universe with any spacial curvature, by calculating the difference between the horizon degrees of freedom and the bulk degrees of freedom in a region of space and applying the proposal (8). The results obtained in this paper together with those of [15, 16] further support the new proposal of Padmanabhan [14] and its modification as (8) and show that this approach is powerful enough to apply for deriving the dynamical equations describing the evolution of the Universe in other gravity theories with any spacial curvature.

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